Antiderivative of $\tan x \sec^2 x$

Compute $\displaystyle\int \tan x \sec^2 x \, dx$ in two different ways:

a) By substituting $u = \tan x$.

b) By substituting $v = \sec x$.

c) Compare the two results.

Antiderivative of $\tan x \sec^2 x$

5/8/25

Compute $\displaystyle\int \tan x \sec^2 x \, dx$ in two different ways:

a) By substituting $u = \tan x$.

b) By substituting $v = \sec x$.

c) Compare the two results.

a) $u = \tan x \Rightarrow du = \sec^2 x \, dx$

$$\int \tan x \sec^2 x \, dx = \int u \, du$$
$$= \frac{u^2}{2} + C$$
$$= \frac{\tan^2 x}{2} + C$$

$$\frac{d}{dx}\left(\frac{\tan^2 x}{2}\right)$$
$$= \tan x \sec^2 x$$

b) $v = \sec x \Rightarrow dv = \dfrac{\tan x}{\cos x} dx$

$$\int \tan x \sec^2 x \, dx = \int v \, dv$$
$$= \frac{v^2}{2} + C$$
$$= \frac{\sec^2 x}{2} + C$$

$$\frac{d}{dx}\left(\frac{\sec^2 x}{2}\right)$$
$$= -\frac{1}{\cos^3 x}(-\sin x)$$
$$= \tan x \sec^2 x$$

c) Both are correct but depending on substitution u, you get a similar trigonometric function as the integral.

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \sec^2 x (1 - \cos^2 x) = \sec^2 x - 1$$

$\therefore$ They differ by a constant 1.